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Natural convection in enclosures with floor cooling subjected to a heated vertical wall

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Abstract

Natural convection in enclosures is extensively investigated due to its importance in many applications, such as heat transfer through double glazing windows, electronic cooling devices, geophysical applications, etc. Two configurations that have been extensively explored in the literature are the differentially heated enclosures and the Rayleigh–Benard problems. In the present work, a different kind of problem is investigated, namely the cross thermal boundary conditions. Three dimensional analyses were performed for an enclosure cooled from below with one vertical wall heated, and the other connecting walls were assumed to be adiabatic. The thermal condition at the ceiling is varied from an adiabatic one to a different degree of heating. The objective of this study is to simulate the comfort provided by floor cooling in a room. For comfort requirements, the interest is on determining the rate of heat transfer and the temperature distribution in the room. Also, the results have importance for other cooling applications such as electronic cooling and natural convection in freezers. Furthermore, the problem is academically interesting for understanding the fundamentals of natural convection. Based on the authors' knowledge, the physics of this problem has not been explored by other people in such a detail. However, the application has been in practice from ancient times.

The predicted results are interesting and have practical applications. For a certain configuration, where strong three dimensional recirculations were predicted, the flow is three dimensional, hence the two dimensional assumption is not valid. Also, it is found that the rate of rate transfer from the floor is a weak function of the investigated parameters, such as Rayleigh number.

1. Introduction

Natural convection in enclosures can be historically classified into three groups: enclosure heated from below and cooled from above (Rayleigh–Benard problem), differentially heated enclosures, and enclosures with cross thermal boundary conditions.

The interest in natural convection in cavities goes back to early 1900. Benard (1900) and Rayleigh (1916) studied the stability of flows in cavities heated from below. It is believed that the driving force in Benard's experiments can mainly be attributed to the surface tension. However, the buoyancy driven flow in a cavity heated from below and cooled from above is referred to as the Rayleigh–Benard problem. The problem of

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an enclosure heated from below has been extensively studied by many researches with different attentions. A review on the mentioned problem can be found in the references [1,2].

Interest on natural convection in differentially heated enclosures started to investigate the rate of heat transfer in double glazing windows, solar collectors and geothermal applications. The earliest work on natural convection in a differentially heated enclosure was found to belong to Batchelor [3]. This problem was extensively studied both experimentally and numerically. Most analytical work done was based on two dimensional assumptions [4–8]. Latter on, three dimensional analyses were performed and it was found that the flow structure is three dimensional due to the lateral boundary layer effects. Also, the effects of thermal boundary conditions on flow and heat transfer were studied [9–14]. However, the average rate of heat transfer is not that sensitive to the lateral connecting walls, i.e., the average Nusselt number predicted by two dimensional assumptions is compared with the one obtained from three dimensional analyses.

Cross thermal boundary conditions are the main focus in the present investigation. The problem is interesting for comfort cooling and heating of buildings. Historically, floor heating dates back to Roman times currently are being employed for achieving comfort just in a few places. Also, floor cooling, obtained by passing cooling fluid through concrete floor slabs has been considered by few researchers [15–18]. Floor cooling and heating is mainly based on radiation exchange between walls and objects. However, natural convection is very important for thermal stratification, which may have unfavorable effects on the thermal comfort. The main problem encountered in floor cooling is the water vapor condensation, provided the floor is cooled below the saturation temperature. This may become a major problem, especially in humid regions.

The problem can be idealized by assuming that the floor is kept at a constant, low temperature, while one of the walls exposed to the ambient is kept at a constant high temperature. Other adjusted rooms are assumed to be at the same conditions as the control room, therefore other vertical walls are assumed to be adiabatic. The ceiling may have different thermal conditions depending on the condition of the second floor. That is, different scenarios were explored, namely adiabatic and different degrees of cooling. Also, the problem has important academically to understand the heat and fluid flow in enclosures with different boundary conditions. The results show very interesting phenomena, three dimensional vortices hanging at the ceiling of the enclosure. It should mention that, the radiative heat transfer is not coupled with convection because air is assumed to be non-participating medium and temperature of heat and cooled boundaries are summed to be known. Therefore, radiative heat flux can be calculated as an additive quantity.

2. Problem definition and governing equations

The problem under consideration is an enclosure filled with air of $Pr = 0.71$, Fig. 1. The dimensions of the rectangular enclosure are L_x, L_y and L_z. The floor is kept at a low temperature, T_c, and the left wall is kept at a high temperature, T_h. Different scenarios were explored by imposing different thermal conditions for the ceiling. These include adiabatic and different rates of heating conditions, which mainly depend on the thermal condition of the roof. For instance, if the room is located on the first floor and the second floor is also using floor cooling, then the ceiling thermal condition depends on the insulation layers as well as on the location of the cooling tubes. The other connecting walls are assumed to be adiabatic. The air was assumed to be Newtonian and incompressible, Boussinesq approximation being valid.

Using the following dimensionless variables: $X = x/L_x$, $Y = y/L_x$, $Z = z/L_x$, $\vec{V} = \vec{v}L_x/\nu$, $P = pL_x^2/\rho\nu^2$, $\Theta = (T - T_0)/(T_1 - T_0)$, where ν is the kinematical viscosity of the fluid and $\vec{v}$ is the velocity vector, the equations governing the conservation of mass, momentum and energy in a non dimensional form can be written as follows:

$$\vec{\nabla}.\vec{V} = 0 \tag{1}$$

$$(\vec{V}.\vec{\nabla})\vec{V} = -\vec{\nabla}P + \nabla^2\vec{V} + \frac{Ra}{Pr}\Theta\vec{k} \tag{2}$$

$$\vec{V}.\nabla\Theta = \frac{1}{Pr}\nabla^2\Theta \tag{3}$$

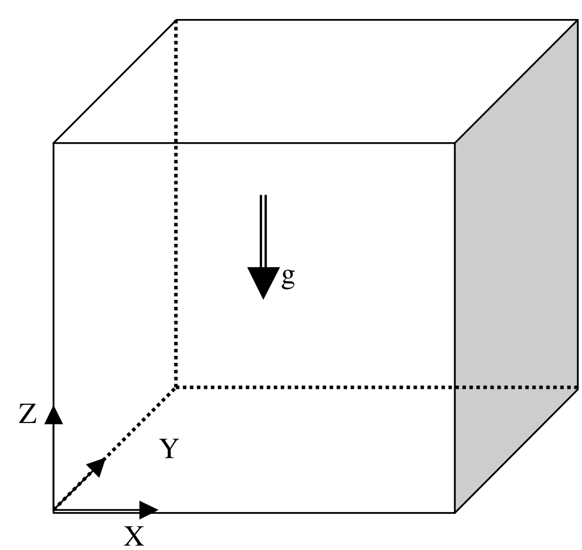

Fig. 1. Schematic diagram of the problem with coordinate system.

where $\vec{k}$ is the unit vector in Z direction, $Ra = (g\beta_T \Delta T L^3)/\nu\alpha$ is the thermal Rayleigh number and $Pr = \nu/\alpha$ is the Prandtl number.

All boundaries are assumed to be rigid, no-slip conditions being imposed for velocities

$$U = V = W = 0 \quad \text{on } X = 0, A_x$$
$$U = V = W = 0 \quad \text{on } Y = 0, A_y$$
$$U = V = W = 0 \quad \text{on } Z = 0, 1 \tag{4}$$

The following boundary conditions are used for the energy equation.

$\Theta = 0$ on $Z = 0$, for $Z = 1.0$ different rates of cooling were imposed, i.e., $\Theta = 0$, 0.2, 0.4, 0.6, 0.8, 1.0 and adiabatic.

$$\Theta = 1 \quad \text{on } X = 0 \quad \text{and} \quad \partial\Theta/\partial X = 0 \quad \text{on } X = A_x$$
$$\partial\Theta/\partial Y = 0 \quad \text{on } Y = 0, 1 \tag{5}$$

The Nusselt number is calculated as $Nu = -\frac{\partial\Theta}{\partial n}$, where n represents a direction normal to a wall. The average Nusselt number is defined as $Nu_{\mathrm{av}} = \frac{1}{A}\int Nu \, \mathrm{d}A$.

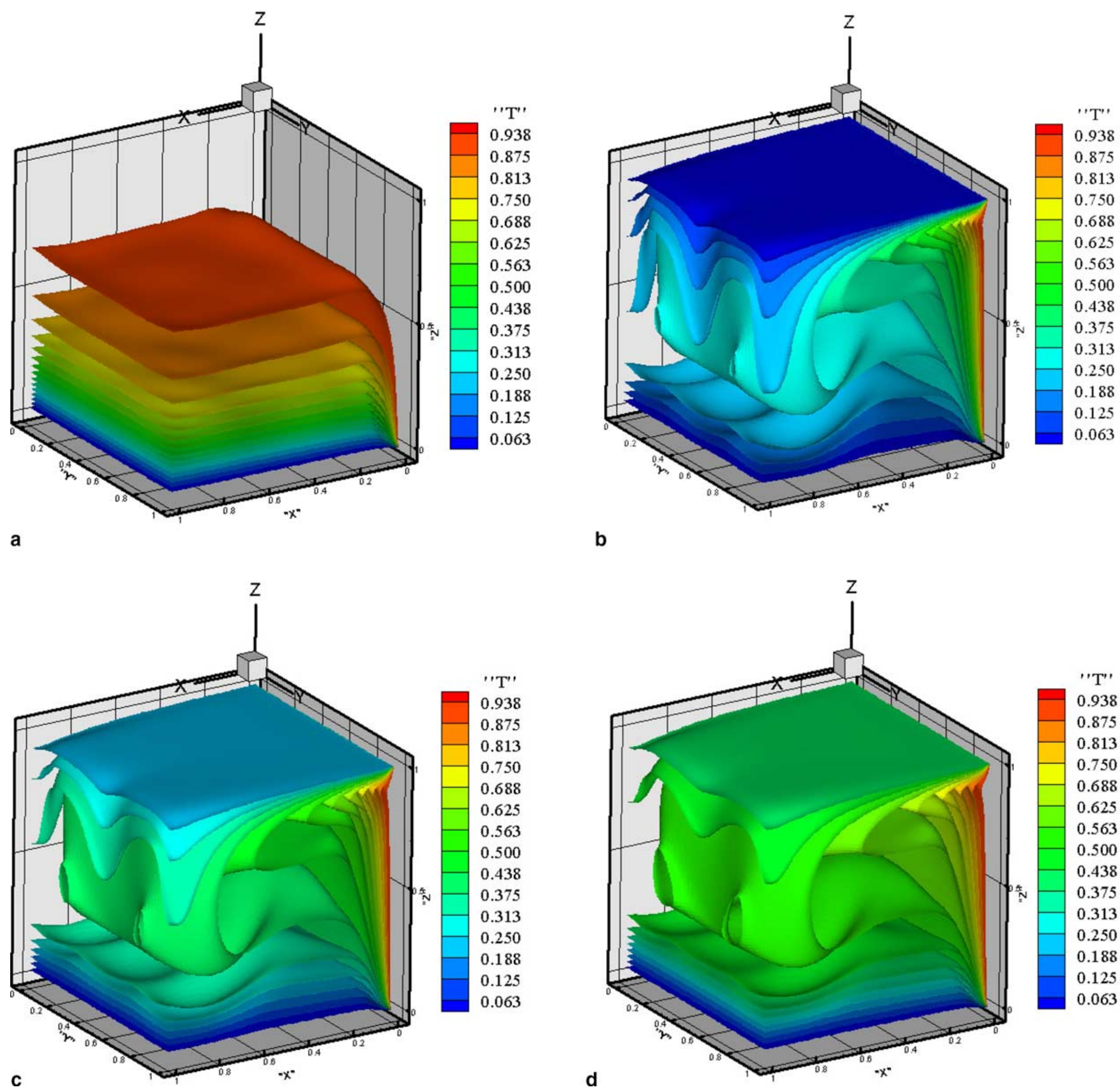

Fig. 2. (a) $Ra = 10^6$, floor cooling, adiabatic ceiling, (b) $Ra = 10^6$, floor cooling, ceiling at $T = 0.0$, (c) $Ra = 10^6$, floor cooling, ceiling at $T = 0.2$, (d) $Ra = 10^6$, floor cooling, ceiling at $T = 0.4$, (e) $Ra = 10^6$, floor cooling, ceiling at $T = 0.6$, (f) $Ra = 10^6$, floor cooling, ceiling at $T = 0.8$ (g) $Ra = 10^6$, floor cooling, ceiling at $T = 1.0$.

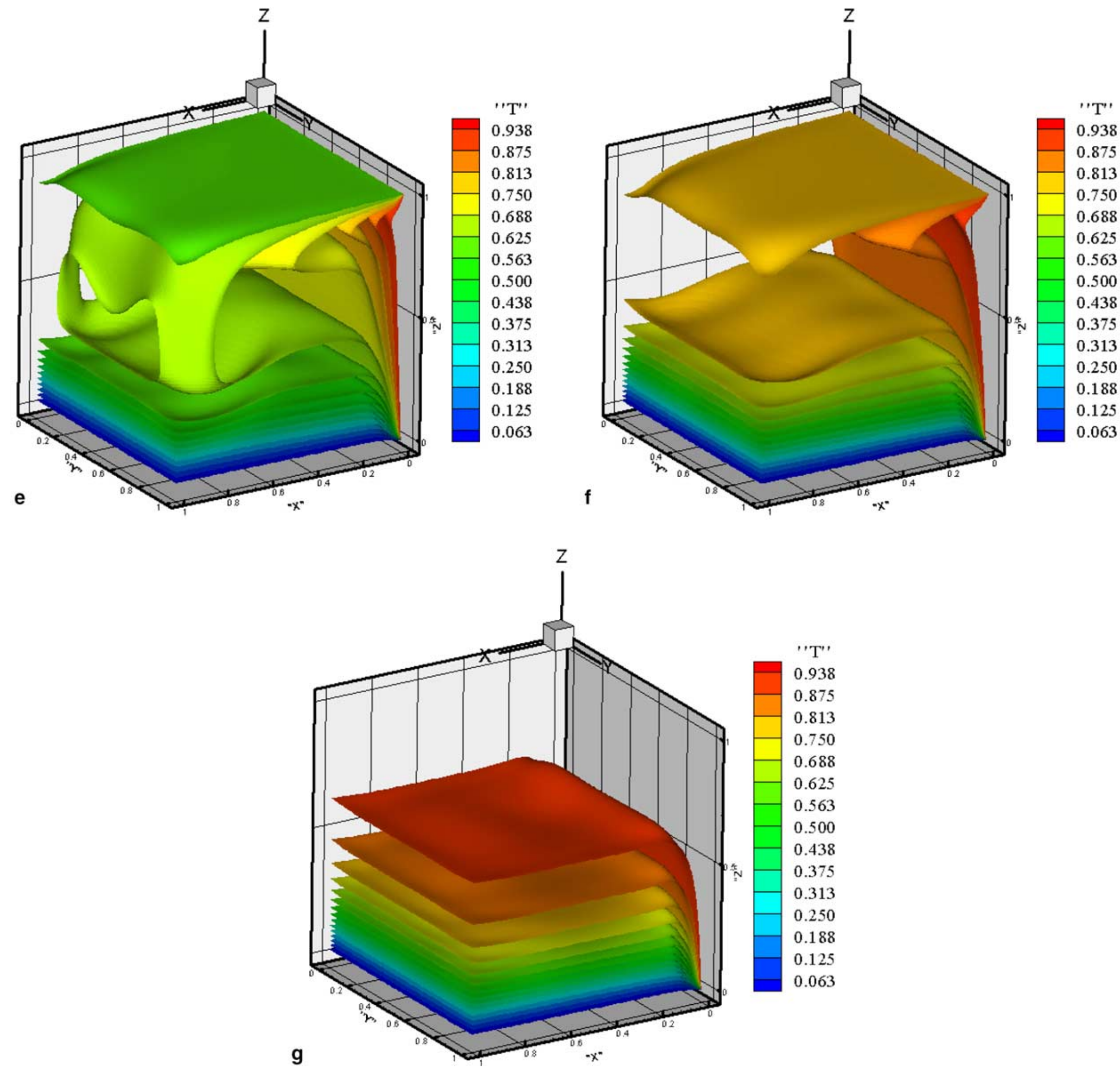

Fig. 2 (*continued*)

3. Numerical method

Eqs. (1)–(5) are discretized using staggered, non-uniform control volumes. In order to minimize the numerical diffusion errors, QUICK scheme is used to approximate the advection terms. The flux limiter known as ULTRA-SHARP is used in order to eliminate the non-physical oscillations inherent in the QUICK scheme. To alleviate the convergence problems, the method is implemented in the solution procedure using the deferred correction approach. SIMPLEC algorithm is used to couple momentum and continuity equations. The momentum equations are solved by applying one iteration of the strongly implicit procedure (SIP). The pressure correction equation is solved iteratively by applying the conjugate gradient (CG) method until the sum of absolute residuals has decreased by a factor of ten. The coefficient matrix, resulting from the discretization of the energy equation is non-symmetric and solved iteratively by employing the Bi-CGSTAB method. SSOR preconditioning is used for accelerating the convergence rates of both CG and Bi-CGSTAB methods. Generally, under relaxation factors of 0.7, 0.7, 0.7, 1.0 and 0.9 were applied to U, V, W, P and T, respectively. For more details on the numerical scheme we referee to the work of Sezai and Mohamad [19].

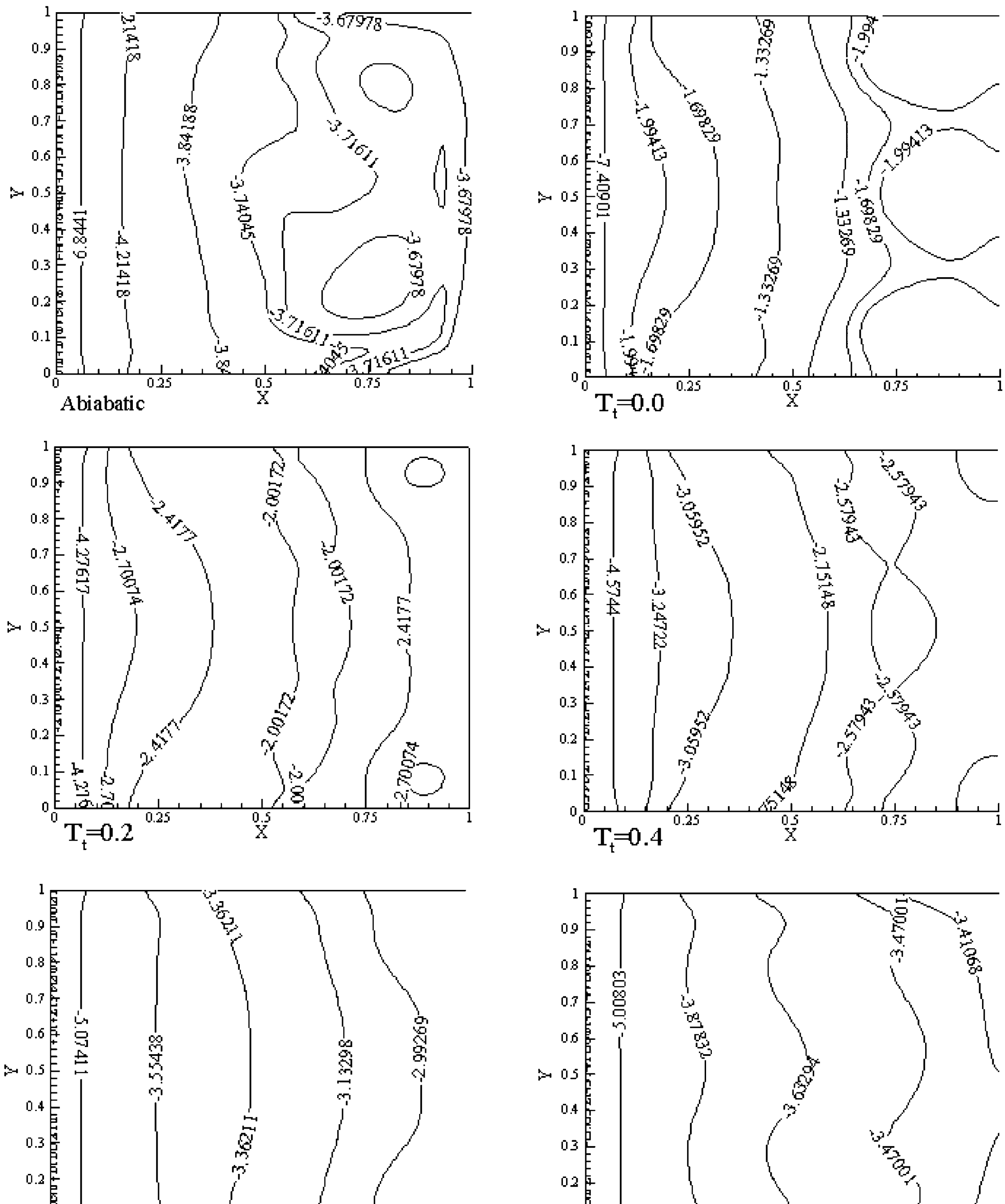

Fig. 3. Floor cooling, Nu at the floor, $Ra = 10^6$, $A_x = 1.0$.

To avoid the excessively high computer times inherent in the solution of three dimensional natural convection problems, the full multigrid method is used to solve the problem, which removes a wider spectrum of wavelengths more efficiently than the single grid methods. The equations are solved by a four level fixed V-cycle

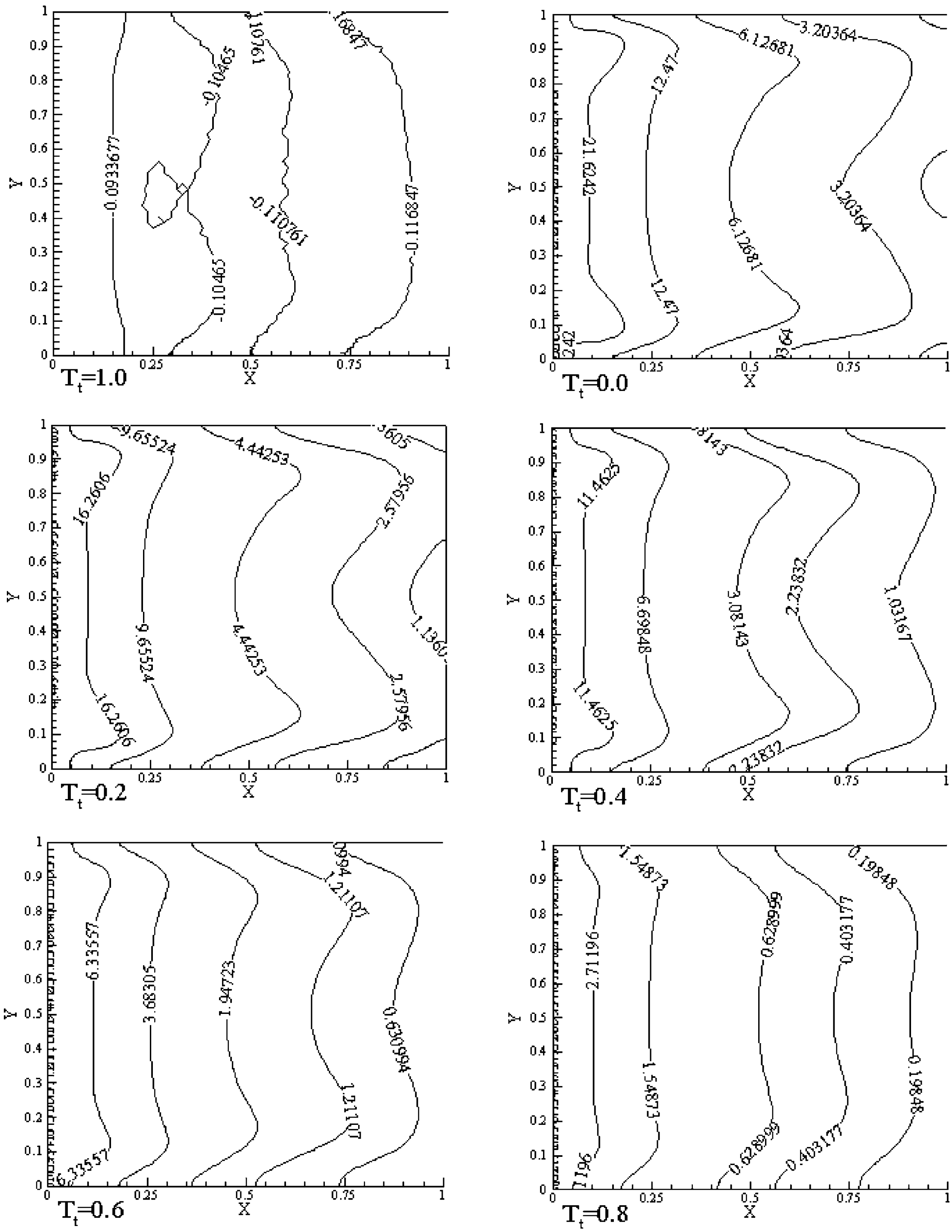

Fig. 4. Floor cooling, *Nu* ceiling, $Ra = 10^6$, $A_x = 1.0$.

procedure starting at the coarsest grid and progressing to the finer grid level. For prolongation operations, tri-linear interpolation is used for all variables. For restriction, the area-weighted average procedure is used for all quantities defined on the control-volume surface such as velocities. The volume-weighted average procedure is

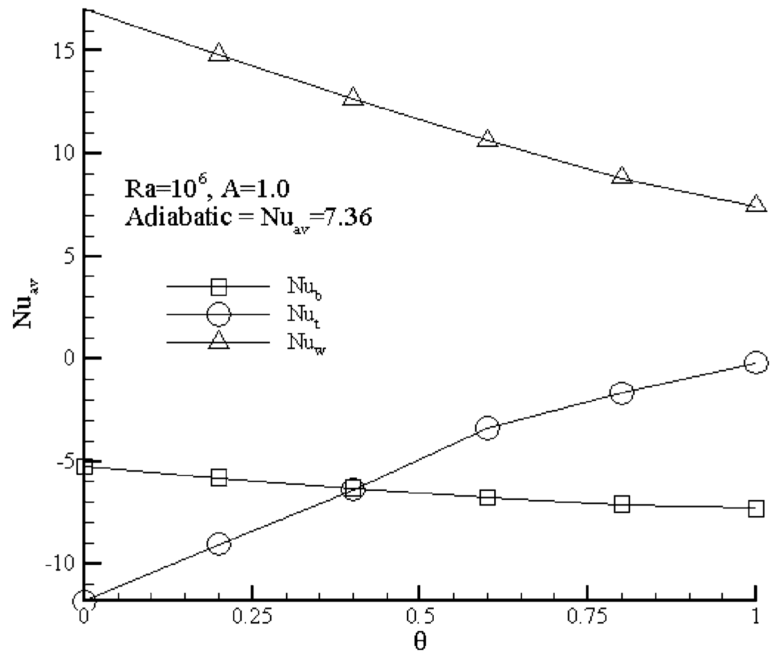

Fig. 5. Average Nusselt number as a function of heating degree of the ceiling, $Ra = 10^6$, $A_x = 1.0$.

adopted for all quantities defined at the control-volume center such as pressure and temperature. The results obtained using $86 \times 86 \times 86$ control volumes were compared with those obtained using $128 \times 128 \times 128$ control volumes and the difference was not noticeable. Therefore, all calculations carried out in this work are based on $86 \times 86 \times 86$ control volumes.

To ensure convergence of the numerical algorithm the following criteria is applied to all dependent variables over the solution domain, $\frac{\sum|\phi^m_{ijk}-\phi^{m-1}_{ijk}|}{\sum|\phi^m_{ijk}|} \leqslant 10^{-5}$, where ϕ represents a dependent variable U, V, W, P and T, the indexes i, j, k indicate a grid point and the index m provides the number of the current iteration at the finest grid level. It is found that 50 iterations is more than necessary to obtain a convergent solution.

4. Results and discussion

Since the interest is on the thermal comfort achieved by employing floor cooling, the temperature and local Nusselt distribution in the enclosure are presented and discussed for Rayleigh numbers of 10^6 and 10^7. The flow became unstable for higher Rayliegh numbers, $Ra > 10^7$. The results are presented for a cubic enclosure and for an enclosure of aspect ratio $A_x = 0.5$.

Fig. 2 shows the temperature field in the cubic enclosure for different thermal boundary conditions at the ceiling of the enclosure: adiabatic, kept at a temperature equal to that of the floor, $\Theta = 0$; different degrees of heating, 0.2, 0.4, 0.6, 0.8; and constant temperature equal to that of the heated wall, $\Theta = 1.0$, respectively and. The value of the Rayleigh number is $Ra = 10^6$.

The flow is thermally stratified in the core of the enclosure for the adiabatic ceiling and for ceiling kept at hot wall temperature condition, $\Theta = 1.0$. The results show a sharp temperature gradient near the floor, which gradually decreases up to a height of about 0.5 (middle of the enclosure). The temperature becomes nearly constant for the upper portion of the enclosure ($Z > 0.5$). The temperature distribution is almost uniform in planes parallel to the floor, except near the heated wall.

An interesting temperature distribution and flow pattern can be noticed for cold ceiling condition, $\Theta = 0$. The flow became strongly three dimensional and falling plumes form at the ceiling descending toward the adiabatic wall. Two transverse elliptical vortices form near the adiabatic wall and their strength decreases near the heated wall. The strength of the transverse vortices decreases as the heating rate of the ceiling increases and completely diminishes when the ceiling temperature is equal to the heated wall temperature.

For $\Theta = 0.6$ a primarily buoyancy driven cell forms. The strength of the cell decreases as Θ increases, and totally diminishes for $\Theta = 1.0$. The flow is thermally stratified when the ceiling temperature is equal to the heated wall temperature.

High temperature gradients are observed near the floor for all investigated conditions.

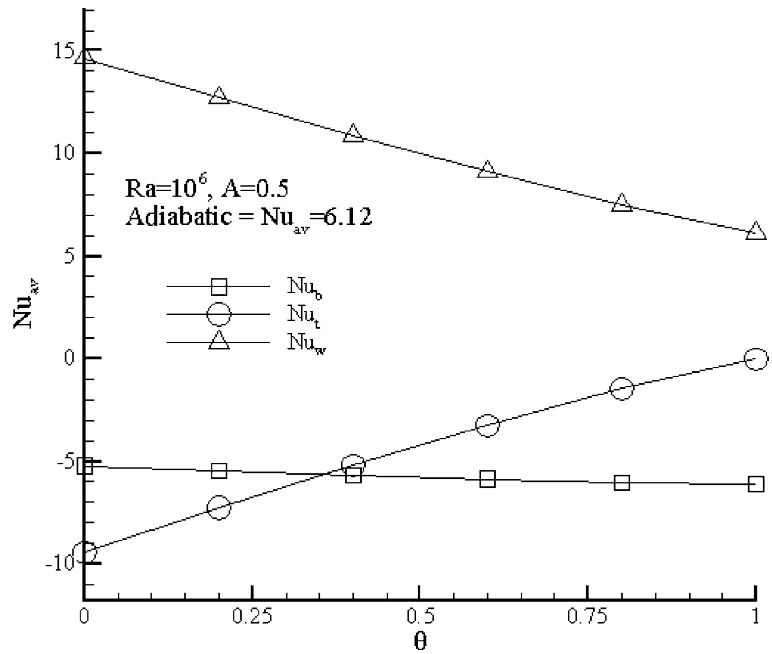

Fig. 6. Average Nusselt number as a function of heating degree of the ceiling, $Ra = 10^6$, $A_x = 0.5$.

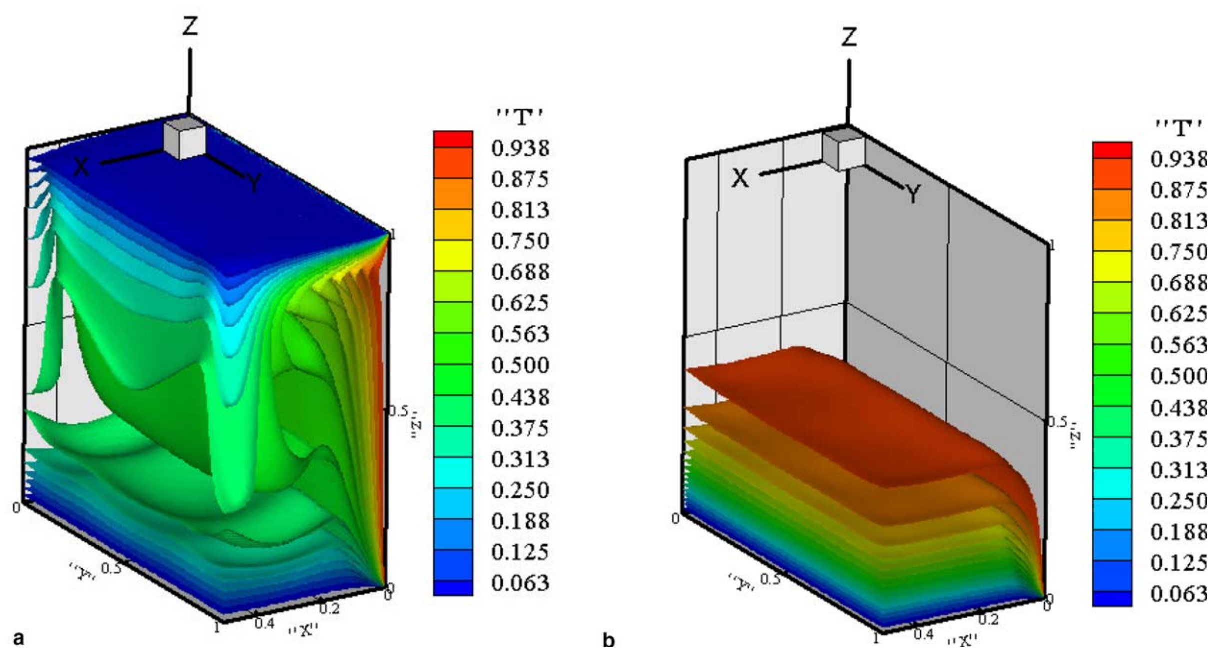

Fig. 7. (a) $Ra = 10^6$, $A_x = 0.5$, floor cooling, ceiling at $T = 0.0$, (b) $Ra = 10^6$, $A_x = 0.5$, floor cooling, ceiling at $T = 1.0$.

Local Nusselt numbers on the floor and on the ceiling are shown in Figs. 3 and 4, respectively, for $Ra = 10^6$. The boundary conditions at the ceiling significantly influence the local Nusselt number at the floor. The local Nusselt numbers along the floor is high near the hot wall and decreases toward the adiabatic wall, which is also supported by the temperature field. This can be explained based on the fact that the thermal boundary layer thickness is thin at the floor, near the hot wall, and increases towards the adiabatic one. The three dimensionality of

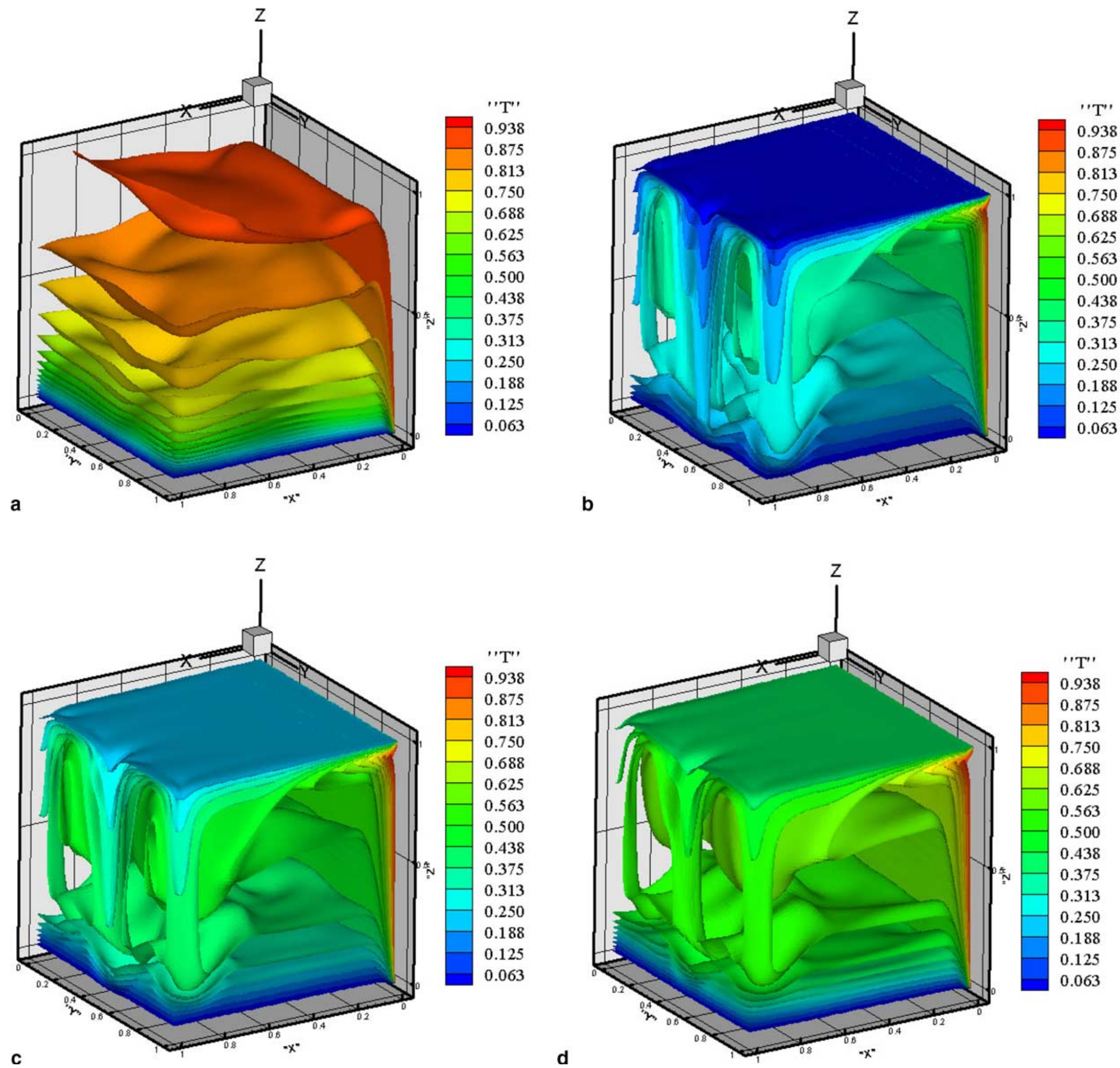

Fig. 8. (a) $Ra = 10^7$, floor cooling, adiabatic ceiling, (b) $Ra = 10^7$, floor cooling, ceiling at $T = 0.0$, (c) $Ra = 10^7$, floor cooling, ceiling at $T = 0.2$, (d) $Ra = 10^7$, floor cooling, ceiling at $T = 0.4$, (e) $Ra = 10^7$, floor cooling, ceiling at $T = 0.6$, (f) $Ra = 10^7$, floor cooling, ceiling at $T = 0.8$ and (g) $Ra = 10^7$, floor cooling, ceiling at $T = 1.0$.

the flow is evident. For $Ra = 10^6$, the Nusselt number at the ceiling shown in Fig. 4 presents a minimum value for heated ceiling and increases as the ceiling cooled. The negative values of Nusselt number indicate that heat is being transferred from the ceiling to the air. The Nusselt number reaches its highest values near the hot wall and decreases toward the adiabatic wall. The three dimensionality of the flow has a significant effect on the local Nusselt number variation in the transverse direction, especially for cold ceiling conditions, $\Theta = 0.0$.

Area-averaged Nusselt numbers on the floor, ceiling, and heated wall are shown Fig. 5 for different ceiling cooling rates and for $Ra = 10^6$. The average Nusselt number on the floor is a weak function of the rate of ceiling cooling. While the Nusselt number on the heated wall decreases significantly as the ceiling cooling rate decreases. For instance, for a cold ceiling the Nusselt number on the wall is about 17 and drops to about 7 for a heated ceiling condition. The average value of the Nusselt number on the ceiling is about 12 for a cold ceiling and reaches almost zero as the ceiling temperature becomes equal to the hot wall temperature.

Similar results are obtained for an aspect ratio of 0.5 ($A_x = 0.5$). Fig. 6 shows average Nusselt numbers

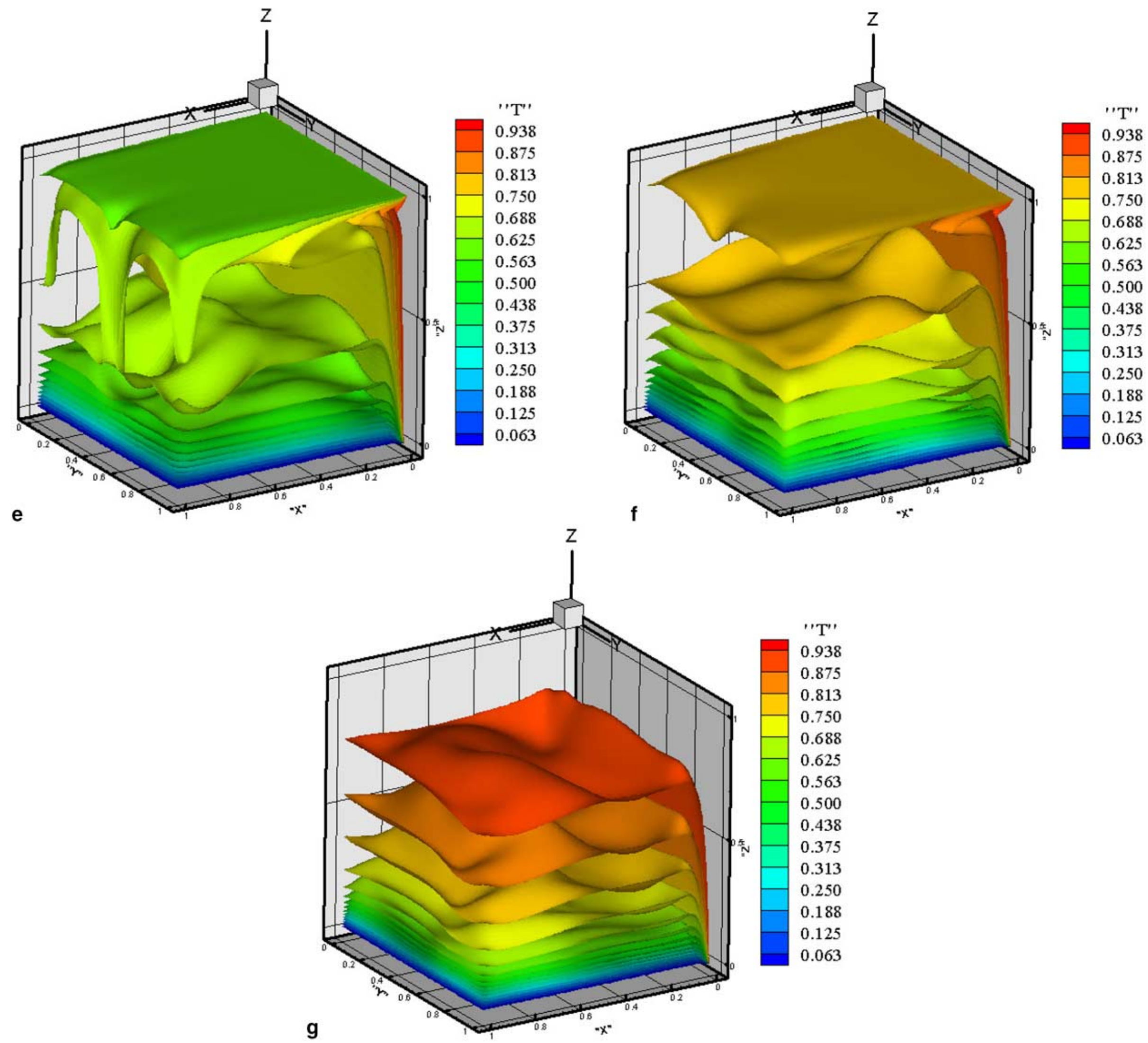

Fig. 8 (*continued*)

as in Fig. 5, except that $A_x = 0.5$. The average Nusselt number on the floor is not so sensitive for the investigated range of aspect ratios. However, the average Nusselt number on the heated wall deceases as the aspect ratio decreases. The same can be said for the ceiling Nusselt number. Also, the temperature profiles for $A_x = 0.5$ showed similar trend, Fig. 7, as for $A_x = 1.0$ in Fig. 2. Hence, there is no need to elaborate on the results of $A_x = 0.5$, which does not add new physical insight.

Results were also obtained for $Ra = 10^7$. The temperature profiles are shown in Fig. 8. The strength of the descending recirculations increases compared with the ones obtained using $Ra = 10^6$, Fig. 2. The three dimensionality of the flow is very clear, even for an adiabatic ceiling condition. The flow exhibits a complex pattern, which is evident from the temperature profiles. For $Ra = 10^6$, the temperature was almost constant in the core of the enclosure for heights greater than about 0.4 and for adiabatic or heated ceiling conditions, Fig. 3, while for $Ra = 10^7$, the flow is stratified and temperature increases with the height for the afore mentioned conditions. For $\Theta = 0$–0.6, the flow descends at the corners and at the center plane along the adiabatic wall. This indicates that three cells form in the enclosure, the strength of these cells decreases as Θ increases. The circulations enhance the mixing process in the core of the enclosure and consequently reduce the temperature gradient in the core of the enclosure.

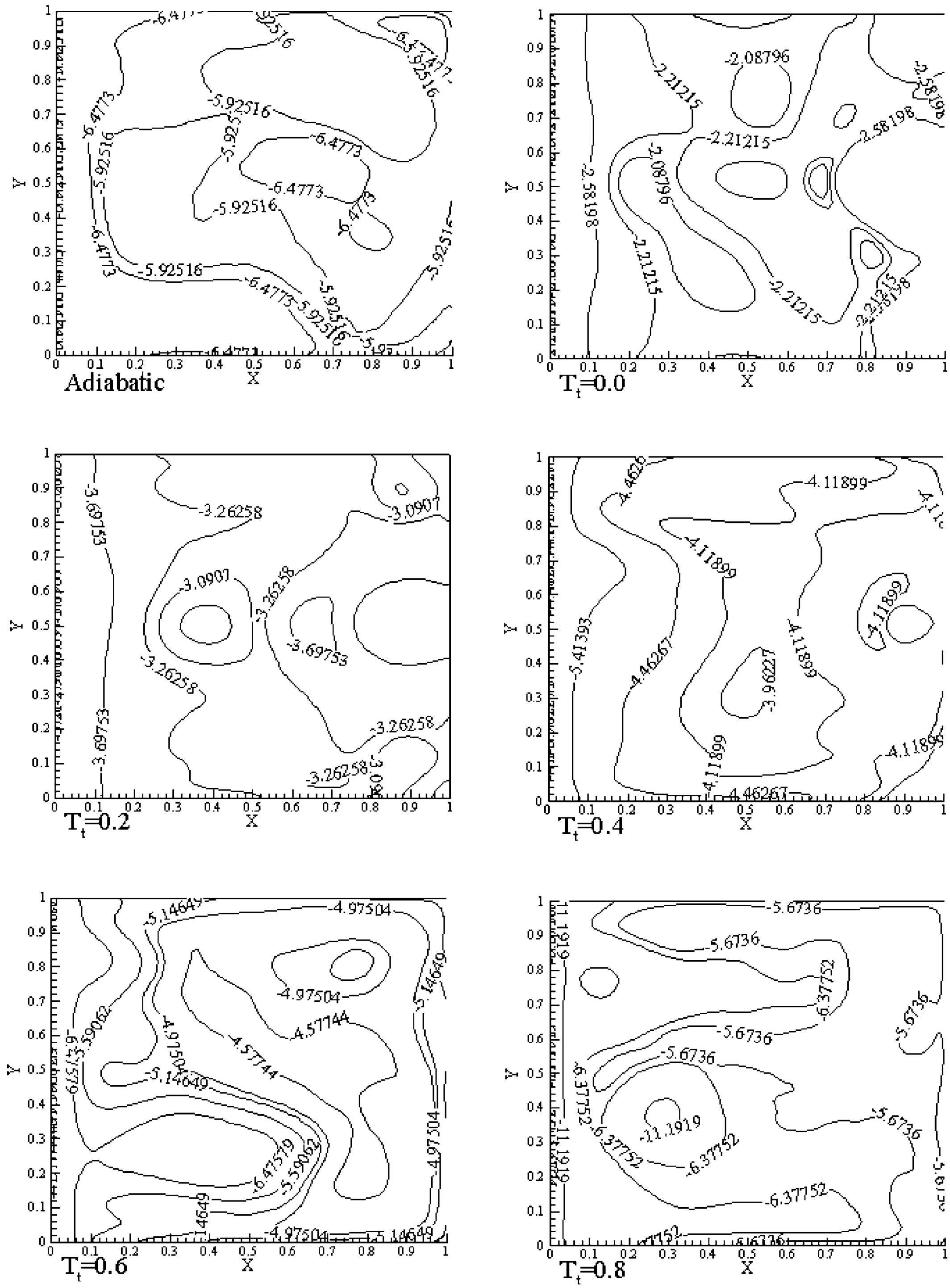

Fig. 9. Floor cooling, $Ra = 10^7$, $A_x = 1.0$, Nu floor.

The local Nusselt number distributions on the floor are very complex due to the complex flow pattern, Fig. 9. Yet, it has higher value near the heated wall compared with the values near the adiabatic wall. The same conclusion can be drawn for the local Nusselt number on the ceiling of the enclosure, Fig. 10.

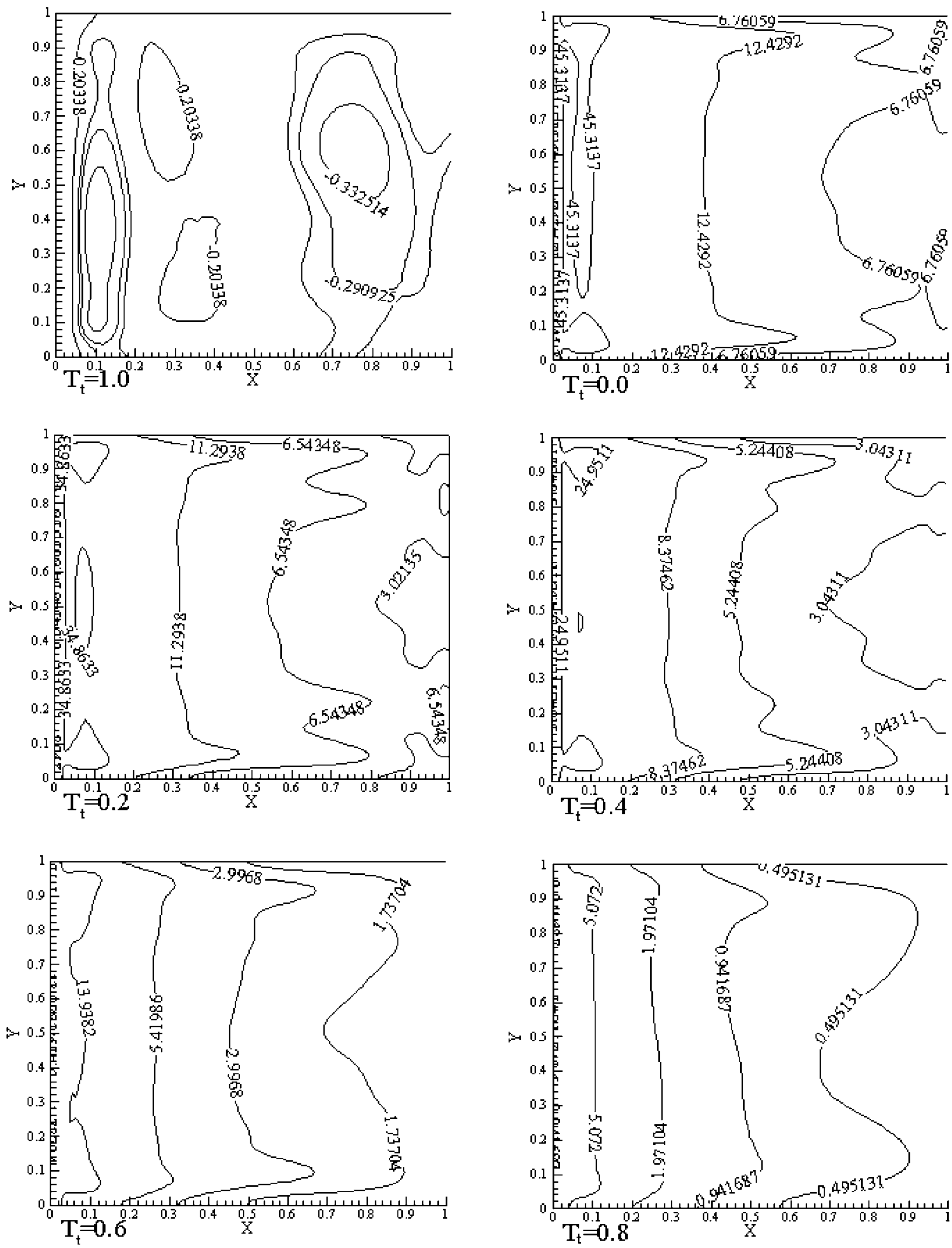

Fig. 10. Floor cooling, $Ra = 10^7$, $A_x = 1.0$, Nu ceiling.

The average Nusselt numbers on the floor are not significantly affected by the *Ra*yleigh number. This can be demonstrated by comparing Fig. 11 with Fig. 5, where

Fig. 11 plots the average Nusselt numbers on the floor, ceiling and heated wall as a function of the rate of ceiling heating for $Ra = 10^7$. The absolute values of the average

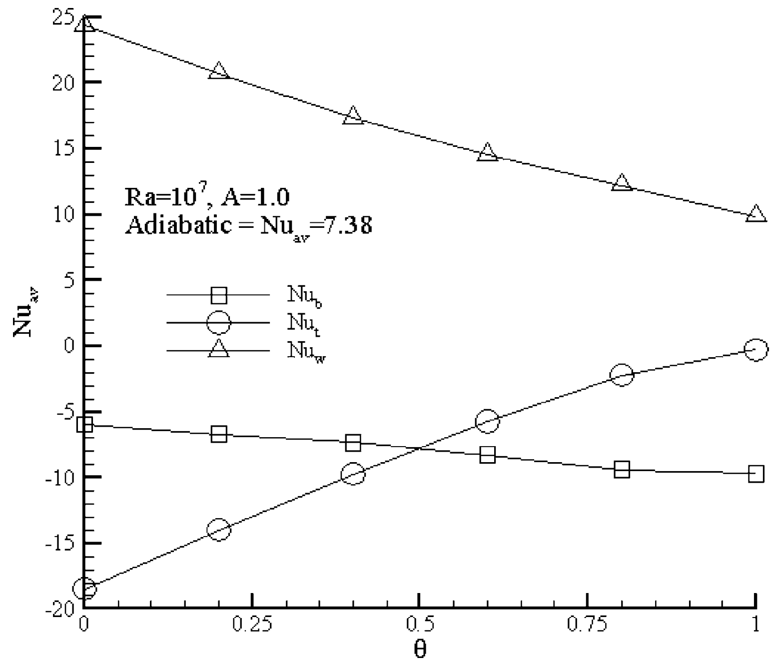

Fig. 11. Average Nusselt number as a function of heating degree of the ceiling, $Ra = 10^7$, $A_x = 1.0$.

Nusselt number on the heated wall and the ceiling increased significantly by increasing the Rayleigh number.

5. Conclusions

Three dimensional natural convection in enclosures subjected to cross thermal boundary conditions was investigated computationally. The flow may be stratified for two specific conditions of the ceiling, namely, adiabatic or heated. The degree of stratification depends on the Rayleigh number. For a cold ceiling there is a possibility of descending traversal recirculations. The strength of these recirculations increases near the adiabatic wall and with the increase in the Rayleigh number. A descending flow at the top corners and at the central plane along the adiabatic wall is evident for $Ra = 10^7$. It is found that the average rate of heat transfer from the floor is almost constant and not a strong function of the Rayleigh number. The rate of heat transfer from the heated wall and ceiling increases as the Rayleigh number increases.

In general, sharp thermal gradients develop near the floor for the entire range of the investigated parameters. Also, the flow exhibits three dimensionality, where transverse recirculations predicted for cold ceiling conditions occur.

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